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On the role of phase transitions for collapsing neutron stars and stellar cores

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Abstract. Phase transitions manifest themselves as strong softening in the equation of state. That influence on the collapse dynamics of neutron stars and stellar cores is considered. The transition of a neutron star to a pion-condensed star can be accompanied by an ejection of a small mass fraction for certain ultradense equations of state. The collapse outcome of stellar cores is not influenced by a phase transition in nuclear matter below nuclear density.

Recently it was pointed out that the possible phase transition from ordinary nuclear matter to nuclear matter with a pion-condensed phase has drastic consequences for neutron stars (Migdal *et al* 1979). If the critical density for the onset of a first-order phase transition in the centre of a stable star is reached, via accretion, cooling or rotational slowing-down, the star becomes unstable (Ramsey 1950, Lighthill 1950, Kämpfer 1981a) and a collapse is triggered. Rough estimations of the gravitational binding energies released during the collapse towards a new stable state give values comparable to typical supernova energies (Migdal *et al* 1979, Kämpfer 1982). However, in realistic neutron star calculations (Haensel and Proszynski 1980, 1981) these energies are reduced, and the possible disappearance of the stable pion-condensed star branch has been shown (Kämpfer 1981b). To be able to identify the collapse of a neutron star to a pion-condensed star as a supernova-like event, the question must be answered whether or not a blowing-off mechanism can be established. This note is devoted mainly to this point.

The very rough model of an incompressible two-fluid configuration shows, after contracting, a damped oscillation of the whole configuration (Migdal *et al* 1979, Kämpfer 1982). In a more realistic model with finite sound velocity, one expects that the outermost shells are in infall motion while the inner part has rebounded. For sufficiently fast infall velocity a sonic point arises, from which an outgoing shock develops. The shock wave, if strong enough, will initiate the blowing-off of mantle layers.

The transition of a neutron star to a pion-condensed star is governed by adiabatic hydrodynamics in dynamic time scales, like the dynamics of stellar core collapse. For studying this transition, a simple equation of state should be used which contains the softening according to pion condensation. In particular, similar to the schematic investigations of stellar core dynamics (Lichtenstadt *et al* 1980, Van Riper 1980), our equation of state is parametrised as

$$P_c = A\rho^{\gamma_A} \qquad \text{for } \rho \leq \rho_1,$$
$$P_c = P_0 + B(\rho - \rho_1) \qquad \text{for } \rho_1 \leq \rho \leq \lambda\rho_1, \tag{1}$$
$$P_c = C\rho^{\gamma_c} \qquad \text{for } \lambda\rho_1 \leq \rho,$$

where the index c indicates the cold component, and P_0 and ρ_1 are the critical pressure and density at which the softening happens. At $\varkappa P_0$ ($\varkappa$ is held fixed at $\varkappa = 1.1$) and $\lambda\rho_1$ the cold equation of state is continued by the γ-law. For the thermal component also a γ-law equation of state is chosen,

$$P_T = E_T\rho(\gamma_T - 1), \tag{2}$$

where E_T is the specific thermal energy. Our calculation scheme follows exactly the difference scheme of Van Riper (1979) in the Newtonian limit as well as formulae (12), (13) of Van Riper (1980).

The critical density ρ_1 for pion condensation is a subject of great uncertainty. With respect to recently claimed shielding effects (Dickhoff *et al* 1981, Meyer-ter-Vehn 1981) $\rho_1 = 1.0 \times 10^{15}$ g cm^{-3} is chosen. For the jump parameter γ values in the range 1.75 . . . 5.0 are assumed to be representative (Migdal 1978, Brown and Weise 1976, Bäckman and Weise 1979).

In figure 1 the world lines of Langrangian matter tracers of a collapsing neutron star model are displayed. The initial cold equilibrium configuration with the central density $\rho_0 = 1.0 \times 10^{15}$ g cm^{-3} is based on the equation of state $P = A\rho^{\gamma_A}$ with $A = 0.6288 \times 10^{10}$ (in CGS units) and $\gamma_A = 1.667$ ($1.4\,M_\odot$ configuration with free Fermi gas adiabatic index). The collapse is initiated by stretching the equilibrium density profile. As a consequence of this procedure the pressure gradient is lowered somewhat and the mass is slightly enlarged. Such a perturbed configuration immediately undergoes a collapse.

As seen in figure 1, a blowing-off of a thin layer happens for $\lambda = 2.0$ and an approximated free Fermi gas thermal index, $\gamma_T = 1.6$, as well as the soft continuation of the equation of state by $\gamma_c = \gamma_A$. This means that in a pure hydrodynamic picture the possibility exists for ejecting a small mass fraction as the result of the collapse of a neutron star to a pion-condensed star. The features of that event are similar to the rebound mechanism (Van Riper and Arnett 1978, figure 1). After passing the zone of the phase transition, the innermost shells are not stopped so abruptly as in the case of a collapsing stellar core. They are compressed further, and, depending on the jump parameter λ and the high-density adiabatic index γ_c, they oscillate more or less. On the inner core part nearly the whole configuration undergoes a homologous oscillation. Only the surface layer is bounced off during the first rebound. A shock wave develops in the outermost part alone. A variation of γ_T in the range 1.2 . . . 1.6 induces only 10% changes of the surface expansion velocities; but for $\gamma_T < 1.2$ it drops quickly below the escape velocity; for γ_T near 1.0 the whole configuration oscillates homologously without any bounce-off effect (cf Berezin *et al* 1981).

The results of model runs with different values of λ and γ_c but fixed γ_T, $\gamma_T = 1.6$, are accumulated in table 1 for the same initial configuration as in figure 1. One sees that mass ejection is hardly possible for small λ and/or large γ_c. The same features hold for models with the critical density $\rho_1 = (0.5$ and $2.0) \times 10^{15}$ g cm^{-3}. These calculations show that the transition of a neutron star to a pion-condensed star (if there is such a thing in nature) can manifest itself as a supernova-like blowing-off

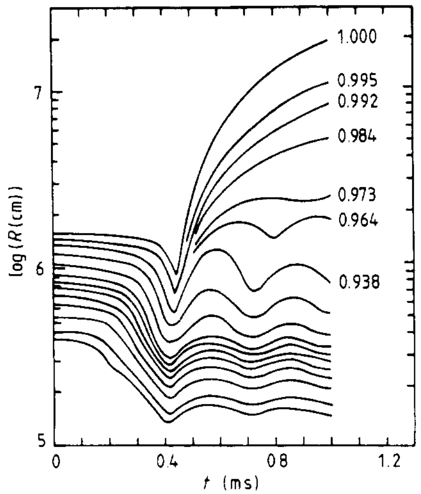

Figure 1. Lagrangian tracers of a collapsing neutron star model. The phase transition happens at $\rho_1 = 1.0 \times 10^{15}$ g cm^{-3} with the jump parameter $\lambda = 2.0$, and the cold adiabatic index is $\gamma = 1.667$ below and above the phase transition. The tracers are labelled by mass fractions inside the shells.

Table 1. The outcome of the transition of a neutron star to a pion-condensed star.

λ	γ_c	Blowing-off[1]	Ejected mass[2]	Energy output[3]
5.0	5.0	Yes	0.015	5.4
5.0	2.5	Yes	0.020	6.7
5.0	1.667	Yes	0.027	10.3
5.0	1.5	Yes	0.027	11.8
2.0	5.0	No	—	—
2.0	2.5	Yes	—	—
2.0	1.667	Yes	0.020	5.7
2.0	1.5	Yes	0.027	11.6
1.75	2.5	No	—	—
1.75	1.667	Yes	0.006	1.2
1.75	1.5	Yes	0.013	5.5

[1] 'Yes' means surface expansion beyond the initial radius, 'No' means oscillations.
[2] Only layers with a velocity above the escape velocity; in units of $M_\odot$.
[3] Only layers with a velocity above the escape velocity; in units of 10^{51} erg.

event. The ejected mass should be observable as filaments surrounding the pion-condensed star. But, if the equations of state with and without pion condensation do not deviate drastically (e.g. there is only a small jump parameter and a stiff ultradense equation of state or large γ_c), as suggested by recent investigations on pion-condensation (cf Meyer-ter-Vehn 1981), the blowing-off disappears. Thus, the proposed special supernova mechanism for identifying the creation of a large pion-condensed core in neutron stars (Migdal *et al* 1979) can operate only for very particular equations of state (which might be unlikely).

In the presently accepted scenario for the stellar core collapse, the pressure is dominated by the electron gas up to nuclear matter density (Bethe *et al* 1979, Brown *et al* 1982). Thus, the softening of the nuclear component in the equation of state below nuclear density according to a phase transition (Lamb *et al* 1981, Friedman and Pandharipande 1981, Barranco and Buchler 1980, Danielewicz 1979, Röpke *et al* 1982) will not influence the collapse dynamics. But, if electrons are captured much more than assumed presently, the neutron matter presure dominates, and the dynamics will be influenced by a phase transition below nuclear density. Bowers *et al* (1975) advocated such a phase transition in neutron matter. For estimating its consequences for the collapse outcome, calculations have been performed using the equation of state (1), (2) for a configuration with initial parameters $\rho_0 = 4.0 \times 10^9$ g cm^{-3}, $A = 9.6 \times 10^{14}$, $\gamma_A = 1.3$ (cf Lichtenstadt *et al* 1980, Van Riper 1980). The Lagrangian tracers are displayed in figure 2 for the following parameters characterising the phase transition: $\rho_1 = 2.7 \times 10^{13}$ g cm^{-3}, $\lambda = 10.0$, $\gamma_c = 2.5$, $\gamma_T = 1.6$. Apart from the other core structure below 1.0×10^6 cm, figure 2 does not show significant differences in comparison with the collapse outcome of the same configuration but without a phase transition. In particular, the ejection energies and masses are not drastically changed. That means there is no considerable influence on the collapse outcome by a phase transition below nuclear density. This holds for other reasonable parameters λ, γ_c and γ_T, too.

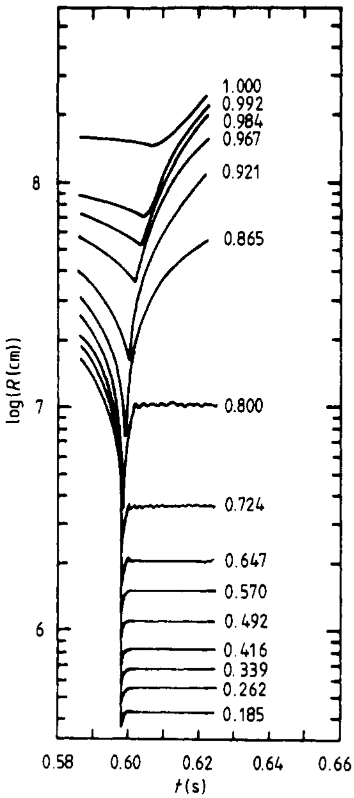

Figure 2. Lagrangian tracers of a collapsing core of a high evoluted star. The cold, high-density adiabatic index is $\gamma_c = 2.5$ after the phase transition at $\rho_1 = 2.7 \times 10^{13}$ g cm^{-3} with $\lambda = 10.0$.

In their relativistic collapse studies, Shapiro and Teukolsky (1980) found that the collapse of certain heavy configurations is stopped in a dynamical time scale by adiabatic pressure build-up at the centre. Therefore, one should conjecture that heating because of a phase transition (e.g. via latent heat of transformation or shock heating because of jumps of the density/pressure/velocity across the phase boundary) gives rise to a similar stopping at a high-entropy pre-black hole state. In this context, the possible relevance of the phase transition to quark matter needs to be investigated. In particular, such a scenario would offer the possibility of a mass ejection by an outgoing shock during the collapse of a heavy configuration to a black hole in dissipative time scales.

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